

MECHANISM OF EMERGENCE OF INTENSE VIBRATIONS OF TURBINES ON THE SAYANO-SHUSHENSK HYDRO POWER PLANT

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It is demonstrated that the level of vibrations of turbines on the Sayano-Shushensk hydro power plant is enhanced by the capability of a compressible fluid to perform its own hydroacoustic oscillations (which can be unstable) in the turbine duct. Based on the previously obtained results of solving the problem of natural hydroacoustic oscillations in the turbine duct and some ideas about turbine interaction with an unsteady compressible fluid flow, results of full-scale studies of turbine vibrations and seismic monitoring of the dam of the Sayano-Shushensk hydro power plant before and during the accident are analyzed.

Key words: *hydroturbine, vibrations, cascade, acoustics, stability, natural oscillations, flutter.*

Introduction. Various possible reasons were given after the accident on the Sayano-Shushensk hydro power plant (HPP), which happened on August 17, 2009. These reasons were analyzed in much detail in [1]. Some versions of the accident are based on the assumption that the turbine was subjected to a certain pulsed high-power action (something like a hydraulic hammer), which exceeded the safety margin of the structure. These versions, however, do not agree with the results of seismic monitoring of the dam of the Sayano-Shushensk HPP before and during the accident. The main reason for the accident is assumed to be the fatigue failure of the fixtures of the turbine cover of the power-generating unit No. 2, which was induced by the high level of turbine vibrations in the standard operation mode. This fact was established by the technical commission investigating the reasons for the accident.

The elevated level of turbine vibrations on the Sayano-Shushensk HPP (as compared with vibrations obtained for its model) was observed in full-scale tests performed back in 1988. It was shown [2, 3] that emergence of such vibrations is caused by the effect of water compressibility on turbine interaction with the unsteady flow, which was ignored in design of the Sayano-Shushensk HPP. The study of this problem was continued in [4, 5]. Based on results of these studies, certain restrictions were imposed on operation of power-generating units of the Sayano-Shushensk HPP. It should be noted, however, that the hydrodynamic problem in [4, 5] (results of these studies were used as a basis for the version described in [1]) was considered in a linear formulation and in a quasi-steady approximation. It is convenient to use the solution of this problem in such a formulation for engineering calculations, but the solution is rather rough and does not explain some qualitative features observed in experiments. In addition, there is a mistake in [4, 5].

In the present work, based on the theory of cascades in an unsteady flow [6–8] and on the laws of aeroacoustics, we study the qualitative effect of nonlinearity and reduced frequency of vibrations on turbine interaction with an unsteady compressible fluid flow, which was partly taken into account in [2, 3], but skipped in [4, 5]. Using the data obtained, we analyze the results of full-scale studies of turbine vibrations and seismic monitoring of the dam of the Sayano-Shushensk HPP before and during the accident.

Sources of Hydroturbine Vibrations. The basic reasons for emergence of hydroturbine vibrations are indicated below (the degree of the influence of each factor depends on the structural features and operation conditions):

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TABLE 1

Results of the Harmonic Analysis of Oscillograms
Obtained in Full-Scale Studies of Vibrations of the Turbine Bearing

$N = 663.2$ MW		$N = 719.8$ MW		$N = 741.6$ MW	
A	f , Hz	A	f , Hz	A	f , Hz
2.271	0.612	6.572	0.934	18.939	1.131
2.238	0.816	29.101	1.401	21.953	1.234
8.964	0.916	8.955	1.867	56.353	1.337
1.863	2.347	10.713	2.334	48.517	1.440
10.880	2.449	7.537	3.268	16.932	1.543
1.874	2.552	—	—	21.823	1.646
3.050	4.899	—	—	9.045	2.468
2.427	51.745	—	—	9.897	2.983

- 1) Mechanical excitation caused by the lack of balance of the turbine rotor;
- 2) Mechanical excitation caused by external factors, for instance, environmental seismic activity, generator vibrations, etc.;
- 3) Hydrodynamic excitation caused by circumferential nonuniformity of the flow induced by:
 - asymmetry of the boundary conditions in the spiral case;
 - perturbation of the flow passing around the guide vanes;
 - perturbation of the flow by the vortex core emanating from the turbine;
 - perturbation of the flow generated by rotating separation, which occurs in certain turbine operation modes;
- 4) Hydrodynamic excitation caused by the unstable character of the flow around the turbine blades, which induces, in particular, unsteady vortex wakes of the Kármán street type;
- 5) Hydrodynamic excitation caused by hydroelastic vibrations of the turbine blades in the flutter mode;
- 6) Hydrodynamic excitation caused by generation of acoustic waves owing to turbine interaction with an unsteady compressible fluid flow and their multiple reflections from various obstacles and open boundaries at the entrance and exit of the turbine duct.

It should be noted that the influence of water compressibility on the hydrodynamic characteristics of turbines (see Sec. 6) was discovered comparatively recently; for this reason, it was not taken into account in design of the Sayano-Shushensk HPP. The anomalous level of vibrations was not observed on the experimental model either. In full-scale studies performed in 1988, however, an elevated level of turbine vibrations was observed in certain modes of its operation; the reason for these vibrations could not be attributed to the action of the known sources of vibrations. The results of a harmonic analysis of oscillograms obtained in full-scale studies of vibrations of the turbine bearing of the power-generating unit No. 10 in forced modes of its operation are summarized in Table 1 and Fig. 1. The activities described in [2–5] were performed to find the reasons for the increased level of these vibrations.

Forced Hydroacoustic Oscillations. The above-considered anomalous vibrations of the turbine arise in turbomachinery operation modes that were prohibited after the studies performed in 1988. Nevertheless, fatigue failure of turbine cover fixtures is caused by forced vibrations of the turbine cover, which are extremely intense on the Sayano-Shushensk HPP even in regimes allowed for operation. An analysis of experimental data on turbine vibrations shows that they reach a high level mainly at low frequencies. Therefore, hydrodynamic forces induced by circumferential nonuniformity of the flow are considered in the present paper as a source of vibrations.

According to [2–5], the increase in the level of turbine vibrations on the Sayano-Shushensk HPP is caused by the influence of fluid compressibility. As a compressible fluid can perform its natural hydroacoustic oscillations in the turbine duct, the unsteady hydrodynamic forces acting on the turbine can be substantially enhanced if the frequency of the hydroacoustic oscillations of the fluid is close to the frequency of the forced vibrations. It turned out that the frequencies of the forced vibrations of the turbines on the Sayano-Shushensk HPP, which are caused by circumferential nonuniformity of the flow, are close to the frequencies of the natural hydroacoustic oscillations in the streamwise direction of the turbine duct. It is impossible, however, to determine the effect of streamwise hydroacoustic oscillations on the forced oscillations of the fluid within the framework of the linear model, where

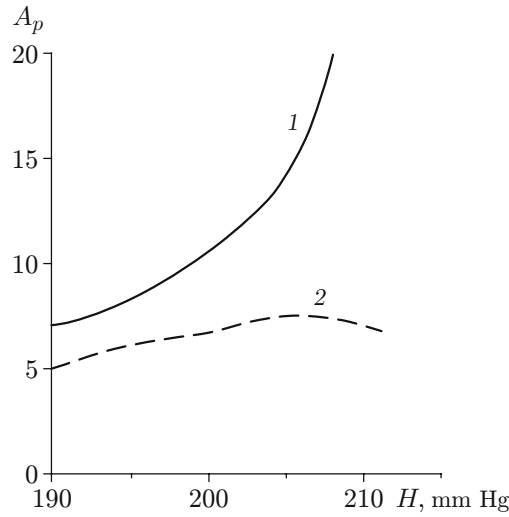


Fig. 1. Maximum amplitude of pressure oscillations A_p versus the fluid flow rate H in the power-generating unit No. 10 of the Sayano-Shushensk HPP: results of full-scale tests (curve 1) and model tests (curve 2).

the duct is considered as a straight tube (see [4, 5]), because there is no mean axial disturbance of the flow in the case of turbine interaction with a circumferentially nonuniform flow. In the streamwise direction of the turbine duct, forced oscillations of the fluid, caused by turbine interaction with a circumferentially nonuniform flow, arise only because the spiral case is asymmetric and the turbine location in this case is shifted relative to the center. The unsteady component of the fluid flow rate, caused by turbine interaction with a circumferentially nonuniform flow in the spiral case, can be approximately determined within the framework of the incompressible fluid model. Solving this problem, we obtain the amplitude function of the velocity potential $\varphi_0^{(n)}$ of oscillatory motion of the fluid, which corresponds to the n th term of the expansion of the specified flow nonuniformity into the Fourier series with respect to the circular coordinate. Substituting this function into the Helmholtz equation for the amplitude function of hydroacoustic oscillations $\varphi^{(n)}$, we obtain

$$\frac{\partial^2 \varphi^{(n)}}{\partial s^2} + k^2 \varphi^{(n)} = f(s),$$

$$f(s) = -k_{0n}^2 \varphi_0^{(n)}(s), \quad k_{0n} = \omega_{0n}/a, \quad \omega_{0n} = n\Omega$$

(s is the arc coordinate of the duct with the beginning at the water conduit entrance, a is the velocity of sound in water, and Ω is the angular velocity of turbine rotation with respect to circular nonuniformity of the flow).

With allowance for boundary conditions, the particular solution $\varphi^{(n)}$ of the thus-obtained nonhomogeneous differential equation determines the forced hydroacoustic oscillations. This solution can be presented as an expansion of acoustic oscillations into a series with respect to the eigenfunctions φ_m :

$$\varphi^{(n)} = \sum_{m=1}^{\infty} \alpha_m^{(n)} \varphi_m,$$

$$\alpha_m^{(n)} = \frac{\eta_m^{(n)}}{\|\varphi_m\|^2} \int_{x=1}^l \varphi_0^{(n)} \varphi_m dx, \quad \eta_m^{(n)} = \frac{k_{0n}^2}{k_{0n}^2 - k_m^2}, \quad k_m = \frac{\omega_m - i\delta_m}{a}$$

(ω_m and δ_m are the frequency and decrement of natural acoustic oscillations).

The unsteady hydrodynamic forces acting on the turbine can be determined by the Cauchy–Lagrange integral for the functions $\varphi_0^{(n)}$ and φ_m . The intensity of these forces is determined by the degree of nonuniformity in the function $\varphi_0^{(n)}$ and by the closeness of the frequencies ω_{0n} and ω_m .

Linear Model of Free Hydroacoustic Oscillations. Let us consider low-frequency hydroacoustic oscillations that may occur in the streamwise direction of the turbine duct. In the one-dimensional approximation, with accuracy to the first order of smallness of the Mach number $M = U/a$ (U is the water flow velocity), the velocity potential of acoustic oscillations $\tilde{\varphi}(s)$ satisfies the wave equation

$$\frac{\partial^2 \tilde{\varphi}}{\partial s^2} - \frac{1}{a^2} \frac{\partial^2 \tilde{\varphi}}{\partial t^2} = 0. \quad (1)$$

Let us find the solution of this equation in the form

$$\tilde{\varphi} = \varphi(s) e^{\lambda t}, \quad \lambda = i\omega + \delta, \quad (2)$$

where ω and δ are the eigenfrequency and decrement of oscillations, respectively.

Substituting Eq. (2) into Eq. (1), we obtain the Helmholtz equation

$$\frac{\partial^2 \varphi(s)}{\partial s^2} + \tilde{k}^2 \varphi(s) = 0, \quad \tilde{k} = k - i \frac{\delta}{a}, \quad (3)$$

where $k = \omega/a$ is the wavenumber.

The solution of Eq. (3) is sought under the following boundary conditions:

$$p(s) = 0 \quad (s = 0, l_1 + l_2), \quad p(s) = -\rho \left(i\omega \varphi + U \frac{\partial \varphi}{\partial s} \right),$$

$$u(l_1 - 0) = u(l_1 + 0), \quad u = \frac{\partial \varphi}{\partial s} \quad (s = l_1),$$

$$\Delta p = p(l_1 - 0) - p(l_1 + 0) = \rho U c_u u(l_1).$$

Here, $p(s)$ is the amplitude function of acoustic pressure, ρ is the density of water, l_1 and l_2 are the lengths of the water conduit and the suction tube, respectively, the spiral case size can be neglected, c_u is the dimensionless complex coefficient depending on the turbine geometry and on the Strouhal number $Sh = \omega b / \sqrt{U^2 + (\Omega R)^2}$, and b and R are the mean chord of the turbine blades and the turbine radius, respectively. Determining the coefficient c_u is a rather difficult problem of the theory of cascades in an unsteady flow [6–8].

The characteristic equation of the posed problem has the form

$$\tan(\tilde{k}l_1) + \tan(\tilde{k}l_2) = iM c_u.$$

Using this equation, with accuracy to the first order of smallness of the Mach number M , we find

$$\omega_m = \frac{m\pi a}{l}, \quad \delta_m = -\frac{U \operatorname{Re}(c_{mu})}{(l_1 + l_2)(1 + \tan^2(km l_1))} \quad (m = 1, 2, 3, \dots). \quad (4)$$

Acoustic oscillations can become unstable at

$$\operatorname{Re}(c_{mu}) < 0. \quad (5)$$

It should be noted that condition (5) is similar to the condition of emergence of the turbine flutter due to axial oscillations

$$x = x_0 \exp(i\omega_m t); \quad (6)$$

if this condition is satisfied, the velocity of oscillatory motion of the turbine is

$$\frac{dx}{dt} = -u(l_1) \exp(i\omega_m t).$$

In this case, according to the theory of cascades in an unsteady flow, the unsteady component of the pressure drop on the turbine can be presented as

$$\Delta p = \rho U_0^2 c_x x / b.$$

The condition of emergence of flutter in elastic structures is the inequality $\operatorname{Im}(c_x) > 0$. In view of Eq. (6), this condition is equivalent to the condition of instability of acoustic oscillations (5).

The analogy described above allows us to conclude that it is necessary to take into account the effect of the Strouhal number on the coefficient c_{mu} in the general case, because this number is one of the main parameters in

the theory of flutter in elastic structures. In particular, it should be noted that the quasi-steady approximation used in [4, 5] has a limited area of applicability in estimating stability of hydroacoustic oscillations. Thus, a quasi-steady model of surging in gas turbomachinery, which is an analog of hydroacoustic instability in the hydroturbine duct, usually satisfies these conditions, whereas the corresponding model [4, 5] for hydroturbines is valid only for sufficiently long water conduits.

Let us consider some general features of emergence of instability of hydroacoustic oscillations, depending on the turbomachinery operation modes, which can occur by analogy with flutter. In non-separated flow around the blades, the classical flutter of turbomachine cascades is known to arise at Strouhal numbers much lower than in separated flow regimes. An approximate solution of the corresponding problem of an unsteady flow of a cascade with an arbitrary geometry can be constructed within the framework of the incompressible fluid model, for instance, by the method developed in [9]. The classical flutter is almost impossible in the case of vibrations of a cascade with one degree of freedom; therefore, the probability of emergence of instability of hydroacoustic oscillations in these regimes is rather low.

The problem of an unsteady flow around spatial hydrodynamic cascades in forced regimes of turbomachinery operation with flow separation from the turbine blades has not yet been solved. Therefore, there are no theoretical estimates of the critical Strouhal number determining the boundary of the region where instability of hydroacoustic oscillations arises in these regimes. Following the above-indicated analogy, however, it is possible to develop experimental methods for estimating stability of hydroacoustic oscillations in turbomachinery.

Effect of Nonlinearity on the Character of Hydroacoustic Oscillations. An analysis of experimental data on hydroturbine vibrations during their operation shows that a number of features of the process observed cannot be explained by the linear theory. Acoustic oscillations in the turbine duct have a more complicated character as well. Let us consider some specific features of the effect of nonlinearity on the character of hydroacoustic oscillations in the turbine duct.

Acoustic waves generated by turbine interaction with an unsteady flow propagate away from the turbine in different directions and are reflected from the open ends of the water conduit and suction tube. The reflected waves are incident onto the turbine cascade from different sides. Owing to diffraction, some of these waves are reflected from the turbine, and some of them pass further [8]. Wave diffraction gives rise to unsteady vortex wakes shed from the turbine blades, with some part of acoustic energy being spent on formation of these wakes. The intensity of these wakes and also the amplitudes and phases of steady acoustic oscillations in the water conduit and suction tube depend on the cascade geometry, basic flow parameters, and Strouhal number. The acoustic oscillations in these regions can be considered as coupled. At a certain ratio of the phases, these oscillations may occur predominantly in one of the regions; therefore, the eigenfrequencies of acoustic oscillations in the duct are determined by the geometry of this region. In the linear theory, the coupling of oscillations in these regions is determined from the condition of matching of the corresponding solutions under the assumption that the spiral case has a zero volume. These conditions, however, ignore the processes described above.

The laws of conservation of mass and acoustic energy are used in [3] to match the solutions in the subdomains. The second law is formulated as a nonlinear relation of the parameters of acoustic oscillations, where the above-mentioned processes are partly taken into account. As a result, the spectrum of eigenfrequencies in the duct predicted by the linear theory was supplemented with a subset of eigenfrequencies corresponding to natural acoustic oscillations in the duct regions:

$$\omega_{jm} = \frac{\pi a}{l_j} \left(\frac{1}{2} + m \right) \quad (j = 1, 2, \quad m = 1, 2, 3, \dots). \quad (7)$$

Turbine interaction with an unsteady flow in forced regimes of turbine operation has an essentially nonlinear character because the negative slope of the steady characteristics of the turbine in these regimes is caused by flow separation. The degree of flow separation on a plane model of a cascade shaped as an airfoil is determined by the separation point location. Oscillations of the incoming flow make this point move along the airfoil chord; hence, the force of hydrodynamic interaction of the turbine exhibits periodical changes. As the fluid is viscous, however, separation development, i.e., separation point displacement, requires a certain time. At high Strouhal numbers, when the period of oscillations is small, the available time may be insufficient for separation development. As a result, the unsteady component of the force of hydrodynamic interaction of the airfoil does not reach a value corresponding to its quasi-steady approximation; the higher the Strouhal number, the lower the value of this

component. At sufficiently high Strouhal numbers, we can assume that the value of δ_m is smaller than the value necessary to overcome damping of oscillations caused by losses that are not taken into account in this work, even if inequality (5) is satisfied.

Comparison of Theoretical and Experimental Results. To compare the theoretically expected elevated level of turbine vibrations with experimental data, we calculated the eigenfrequencies of hydroacoustic oscillations in the ducts of Sayano-Shushensk HPP turbomachinery by Eqs. (4) and (7). Substituting the values $l_1 = 241$ m, $l_2 = 28$ m, and $a = 1350$ m/sec into these equations and making corrections for the open ends of the channels and the spiral case size $D = 6.77$ m, we obtain the expressions

$$f_m = 2.41m, \quad f_{1m} = 2.81(1/2 + m), \quad f_{2m} = 19(1/2 + m) \quad (m = 1, 2, 3, \dots). \quad (8)$$

(The eigenfrequencies of hydroacoustic oscillations f are measured in hertz.)

The frequencies of forced vibrations f_n caused by circumferential nonuniformity of the flow are determined by the turbine rotation frequency $\Omega_0 = 2.38$ Hz and by the vortex core precession $\Omega_v = 0.5$ – 1.5 Hz.

Expanding the flow nonuniformity into the Fourier series with respect to the circular coordinate and taking into account that the angular velocity of turbine rotation with respect to the vortex-core-induced nonuniformity is $\Omega = \Omega_0 - \Omega_v$, we obtain

$$f_{0n} = \Omega_0 n, \quad f_{v,n} = (\Omega_0 - \Omega_v) n. \quad (9)$$

The results of the harmonic analysis of oscillograms obtained in full-scale studies of vibrations of the turbine bearing of the power-generating unit No. 10 of the Sayano-Shushensk HPP in the operation mode with $N = 663$ MW and in forced modes of its operation with $N = 719.9$ and 741.6 MW. It follows from Table 1 that the non-forced regimes display forced vibrations of the turbine with the frequency close to the rotation frequency and with the frequency, apparently, close to the vortex core precession frequency. Forced regimes involve intense vibrations with the eigenfrequency of hydroacoustic oscillations f_{11} , which was obtained with allowance for process nonlinearity. An analysis of the maximum amplitudes of pressure oscillations as functions of the fluid flow rate in forced operation modes (see Fig. 1) allows us to conclude that instability of hydroacoustic oscillations appears in this case. At the same time, the pressure oscillations in model tests (dashed curve) are almost independent of the fluid flow rate. This conclusion agrees with the above-made assumptions about the influence of the Strouhal number on turbine interaction with an unsteady flow, because the Strouhal number in the model tests was higher than the corresponding full-scale value by an order of magnitude.

Figures 2 and 3 show the amplitudes of dam vibrations averaged over a certain period of time with a rather wide spectrum of frequencies, which were recorded by sensors of the Cheremushki seismic station placed near the Sayano-Shushensk HPP.

Figure 2 shows the amplitudes of vibrations averaged over the period $T = 100$ sec, which were obtained not long before the accident (A_x , A_y , and A_z are the amplitudes of vibrations in the directions from north to south, from east to west, and along the turbine axis, respectively). Forced vibrations with frequencies (9) can be seen at $\Omega_0 = 2.38$ Hz and $\Omega_v = 0.5$ – 1.5 Hz. The level of these vibrations is rather high, obviously, because their frequencies are close to the frequencies of natural hydroacoustic oscillations (8). Thus, an apparent reason for fatigue failure of the turbine cover fixtures is the increase in the level of vibrations during turbomachinery operation.

The black-and-white copy of the record of vibrations, which was taken during the accident of the power-generating unit No. 2, is shown in Fig. 3 (the time indications correspond to the Greenwich scale). The vertical light bands correspond to instants of recording interruption. It is seen that the accident involves intense vibrations, mostly with frequencies equal to the frequencies f_m of the first five modes (8) of natural hydroacoustic oscillations. Obviously, instability of hydroacoustic oscillations occurred in that case, because the situation with high-intensity forced vibrations for all modes is impossible. This idea is indirectly supported by the experimental data shown in [1, Fig. 22], which contain information that the power of the power-generating unit No. 2 at that period corresponded to the forced regime of its operation. The results in Fig. 3 also confirm the assumption that no instability of acoustic oscillations arises at high Strouhal numbers ($m > 6$). At $t > 00$ h 35 min, when the power-generating unit No. 2 was apparently already destroyed, there appeared intense vibrations [see Eq. (8)] with frequencies f_{06} , f_{07} , and f_{14} corresponding to natural hydroacoustic oscillations in other power-generating units. It follows from the dependences plotted in Fig. 3 that the so-called “capture” of frequencies, owing to nonlinearity of self-sustained oscillations, occurred in a few seconds.

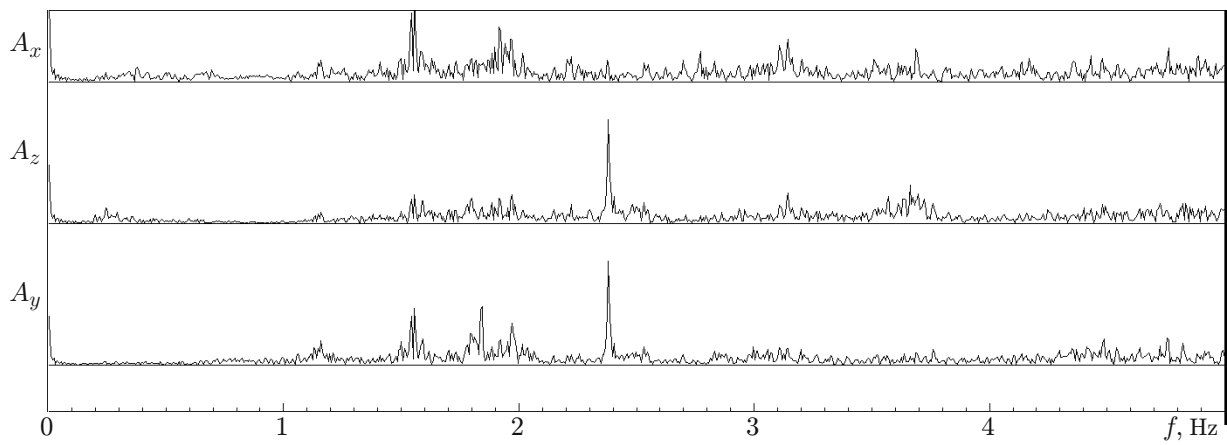


Fig. 2. Records of vibrations from the seismic station sensors on the dam of the Sayano-Shushensk HPP before the accident.

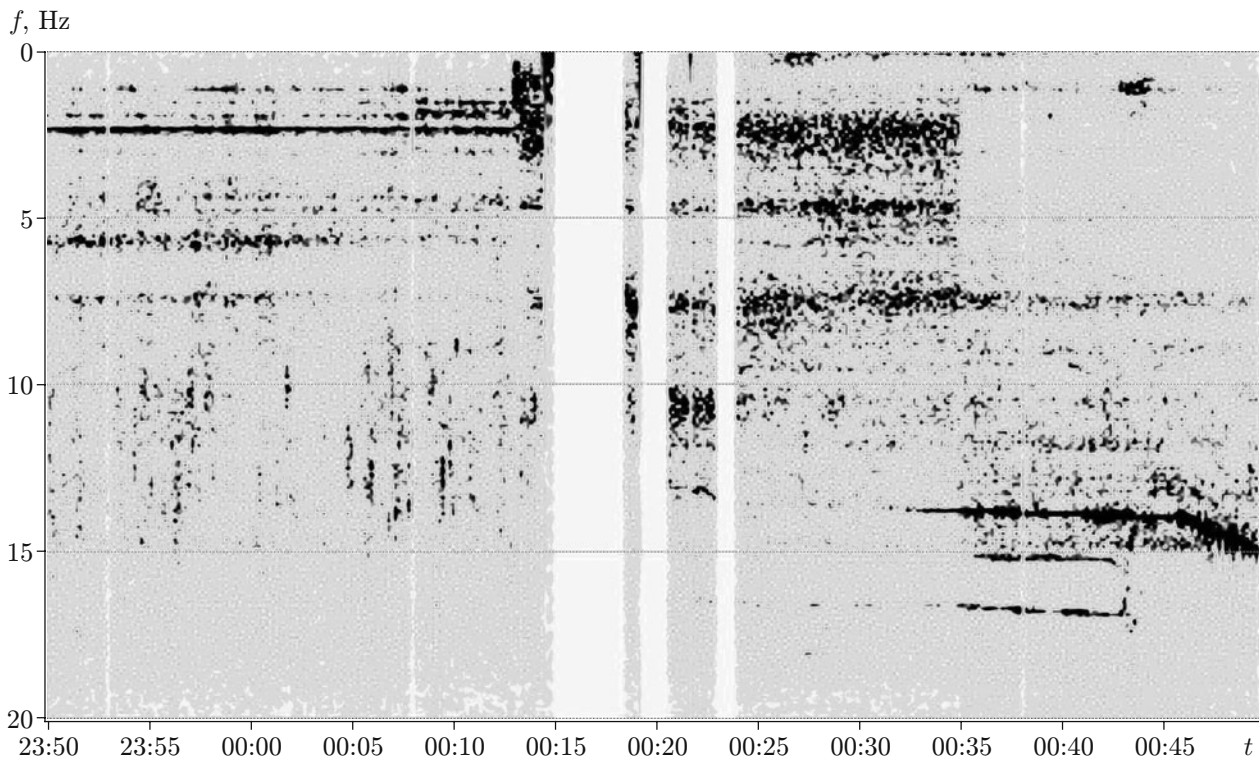


Fig. 3. Records of vibrations from the seismic station sensors on the dam of the Sayano-Shushensk HPP during the accident.

Approximately one minute before the accident, whose starting point was breaking away of the cover of the power-generating unit No. 2, the seismogram shows low-frequency vibrations whose intensity is greater by an order of magnitude than the corresponding value in the standard operation mode of the turbine. Figure 4 shows the time evolution of the amplitude of these vibrations. This process can be assumed to be initiated by failure of sealing of the cover of the power-generating unit No. 2, which occurred because of fatigue failure of its pin fixture. Assuming that the cover was not broken away immediately, we can state that these intense vibrations are related to the dynamics of the cover break-away process accompanied by insignificant impacts on the cover base. As a result, wave motion could arise in the water conduit, which was recorded in the seismogram as oscillations of a rather high

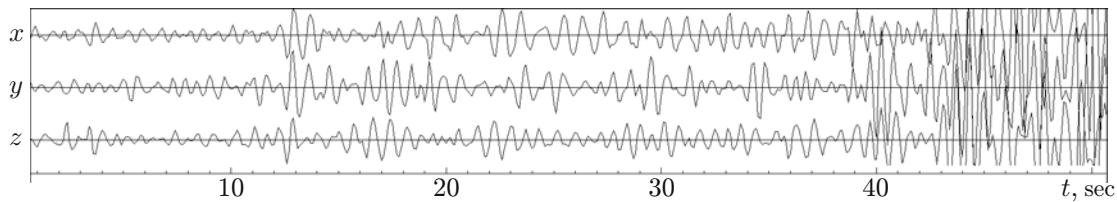


Fig. 4. Records of vibrations from the seismic station sensors on the dam of the Sayano-Shushensk HPP directly before the accident.

intensity. The time period between the instants corresponding to the maximum and minimum amplitudes of these oscillations equals the time needed for hydroacoustic waves to pass from the turbine to the water conduit entrance and back.

Conclusions. Unsteady hydrodynamic forces acting on turbine blades during its interaction with a nonuniform flow depend substantially on whether the frequencies of forced vibrations of the turbine under the action of these forces are close to the eigenfrequencies of hydroacoustic oscillations of the compressible fluid in the turbomachine duct. Instability of natural hydroacoustic oscillations may appear in forced modes of turbomachinery operation, which involves flow separation from turbine blades.

The level of vibrations of Sayano-Shushensk HPP turbines under full-scale conditions is increased over the level of vibrations of the test model in accordance with the above-indicated mechanism. The amplitude–frequency characteristics of dam vibrations obtained by its seismic monitoring during the accident are in good agreement with the parameters of hydroacoustic self-sustained oscillations in the turbine duct.

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